The Thirty-Sixth Annual
Eastern Shore High School Mathematics Competition

November 14, 2019

Individual Contest Exam

Instructions

There are twenty problems on Contest

1. Solve $\frac{e^x + e^{-x}}{2} = 1$.

   a. no solution b. $x = 0$ c. $x = 1$ d. $x = 0; 1$ e. $x = \ln(2)$

2. If $x = \sqrt[5]{-37}$, then which of the following must be true?
   a. $\sqrt{-x} > 2$
   b. $x > -2$
   c. $x^2 < 4$
   d. $x^3 < -8$
   e. $x^4 > 32$

3. An airplane flies 165 miles from point A in the direction 125° and then travels in the direction 245° for 80 miles. Approximately how far is the airplane from A?

   a. 110 miles b. 127 miles c. 143 miles d. 155 miles e. 170 miles

4. Which pair of equations represents two successive vertical asymptotes of the graph of $f(x) = \cot(2x - \quad)$?

   a. $x = -\frac{}{4}$; $x = \frac{}{4}$

   b. $x = 0$; $x =$

   c. $x = \frac{}{2}$; $x =$

   d. $x = 0$; $x = 2$

   e. $x = 0$; $x = 1$

5. Consider the system of inequalities:

$$\begin{cases} x > 0 \\ y \geq 0 \end{cases}$$

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13. The dimensions of rectangle $MATH$ are $2\sqrt{2}$ units and $\frac{15\sqrt{2}}{2}$ units. A marble is released from the midpoint of $\overline{MH}$ and strikes $\overline{MA}$ at an angle of 45° and is reflected across the room to the opposite wall and so on until it strikes $\overline{AT}$. Note: rectangle $MATH$ is not drawn to scale.

What is the length, L, of the marble's path when it strikes $\overline{AT}$?

a. 12 units $\leq L <$ 13 units
b. 13 units $\leq L <$ 14 units
c. 14 units $\leq L <$ 15 units
d. 15 units $\leq L <$ 16 units
e. 16 units $\leq L <$ 17 units

14. When this block of Python code is executed, what is the output?

a. 0 b. 6 c. 16 d. 34 e. 68

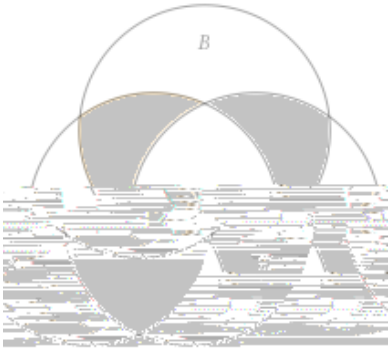
15. Define a sequence a_n, such that the first term of the sequence is $a_1 = 5^3$, and the rest of the sequence is generated using the rule $a_n = 5^{7-4n}$, for $n \geq 2$.

The first three terms of a_n are 5^3, 5^{-1}, and 5^{-5}. What is the sum of all terms of this sequence?

a. $\frac{5^3}{5^4 - 1}$ b. $\frac{5^4}{5^4 - 1}$ c. $\frac{5^7}{5^4 - 1}$ d. 5^4 e. $\frac{5^{12}}{5^4 - 1}$

16. A randomly-chosen 20-year-old man has an 89% chance to live until

17. In this problem, U is a set and for any subset X of U, X^0 represents the complement of X. In the figure below, what does the shaded region represent?



a. $A \setminus B \setminus C$
b. $(A \setminus B \setminus C)^0$
c. $(A \setminus B) \cap (A \setminus C) \cap (B \setminus C)$
d. $(A \cap B) \setminus (A \cap C) \setminus (B \cap C)$
e. $(A \setminus B \setminus C^0) \cap (A \setminus C \setminus B^0) \cap (B \setminus C \setminus A^0)$

18. How many positive integers less than 2019 are a multiple of 20 or 19?

a. 201 b. 202 c. 203 d. 204 e. 205

19. Consider an infinite stack of bricks, in which the bottom brick has a mass of 100 grams, the brick on top of it has a mass of 20 grams, the brick on top of the second has a mass of 4 grams, and each successive brick has a mass $\frac{1}{5}$ of the brick under it. What is the total mass of all these bricks?

a. 120 b. 125 c. 130 d. 135 e. 140

20. A middle school student forgot her locker key password. The only thing she remembered was that none of the four digits (0 through 9) were repeated. To find out the code, she decided to try all possible arrangements of 4-digit codes. Assume it takes exactly 4 seconds to try each code. If the student took the maximum amount of time to decode the lock, the number of hours, H, until the lock was decoded satisfies which of the following?

a. $H < 4{:}5$ b. $4{:}5 \leq H < 5{:}5$ c. $5{:}5 \leq H < 6{:}5$ d. $6{:}5 \leq H < 7{:}5$ e. $H \geq 7{:}5$